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Solution to crack problem 61.

Since the satellite is geosynchronous, hence the angular velocity of the satellite around the earth will be the same as the angular velocity of the earth about its own axis.

Let it be ω

$$\omega = \frac{v_0}{r_0}$$

Also, the gravitational force of attraction between the earth and the satellite is responsible for the centripetal acceleration of the satellite.

$$\Rightarrow \frac{GMm}{r_0^2} = \frac{mv_0^2}{r_0}$$

$$\Rightarrow v_0 = \sqrt{\frac{GM}{r_0}} = \sqrt{\frac{gR^2}{r_0}} \qquad (Answer\, to\, (2))$$

$$\Rightarrow \omega r_0 = \sqrt{\frac{gR^2}{r_0}}$$

$$\Rightarrow r_0 = \left(\frac{gR^2}{\omega^2}\right)^{1/3} \qquad (Answer\, to\, (1))$$

Now, angular momentum,

$$L_0 = mv_0 r_0 = \frac{mgR^2}{v_0} \qquad (Answer\, to\, (3))$$

Now, the mechanical energy,

$$E_0 = K_0 + U_0 = \frac{1}{2}mv_0^2 + \left(-\frac{GMm}{r_0}\right) = -\frac{1}{2}mv_0^2 \qquad (Answer\, to\, (4))$$

Given, $l = \dfrac{L^2}{GMm^2}$

Here (after the rocket is launched),

$$L = mv_0 r_0 + m\Delta v(0) = mv_0 r_0$$

Thus,

$$l = \frac{L^2}{GMm^2} = \frac{(mv_0 r_0)^2}{GMm^2} = \frac{v_0^2 r_0^2}{GM} = r_0 \quad (From\ answer\,(2))$$

$$\Rightarrow l = r_0 \qquad\qquad (Answer\,(5)$$

Now, $e = \left(1 + \dfrac{2EL^2}{G^2 M^2 m^3}\right)^{1/2}$

$$Here,\ E = -\frac{GMm}{r_0} + \frac{1}{2}m\left(v_0^2 + (\Delta v)^2\right) = \frac{1}{2}m\left((\Delta v)^2 - v_0^2\right)$$

$L = mv_0 r_0.$

On solving we get, $e = \left(1 + \dfrac{\left((\Delta v)^2 - v_0^2\right)r_0}{GM}\right)^{1/2} = \left(1 + \dfrac{\left((\Delta v)^2 - v_0^2\right)}{v_0^2}\right)^{1/2} = \dfrac{\Delta v}{v_0}$

Answer to (6)

We can assume that Δv is very small than v, as it is produced due to the launching of a small rocket and thus,

0<e<1 , And so the trajectory of the satellite will be an ellipse.
